

Massless black holes and charged wormholes in string theory

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We present the zero mass black holes and charged Einstein-Rosen bridges (wormholes) that arise from the five parameters dyonic black hole solution of the Einstein-Maxwell-dilaton theory. These massless black holes exist individually in spacetime, different from the known massless solutions which come in pairs with opposite signs for their masses. By imposing appropriate boundary conditions the massless solution can be nonextremal, extremal or a naked singularity. The nonextremal and extremal massless solutions allow the bridge construction, and from them we obtain the first analytical charged Einstein-Rosen bridge satisfying the null energy condition ever found.

Keywords: Massless black holes, wormholes.

I. INTRODUCTION

Zero mass black holes called a lot of attention after Strominger proposed that they were needed to explain the conifold singularities in the low energy theory describing the moduli space of Calabi-Yau vacua of type II string theory [1]. Black holes in string theory with zero ADM mass exist [2–5]. These ones are solutions to the low energy effective theory, but their relation to the massless black holes proposed by Strominger is still unclear, since the latter arise at the conifold singularities, exactly when the low energy theory is singular and the semiclassical description breaks down.

The solution in [2] is stable, since they have the minimal mass allowed by supersymmetry. All the others [3–6] are actually obtained by considering composite supersymmetric pairs (or several pairs) of extremal black holes with masses with "opposite signs" [7]. The classical and quantum instabilities of these pairs are healed by coupling the theory with several scalar fields, which interact with their own scalar charges. These composite objects with masses having opposite signs provide a model for massless black holes, but their relation to the solution of [2] is still unknown. Moreover, there is no way to achieve zero mass limit of known non-extremal individual black holes. This violates the cosmic censor or introduces singular fields, as is the case of the Reissner-Nordström (RN) or the dyonic solution for the Einstein-Maxwell-dilaton (EMD) solution with four independent parameters [8].

The massless black holes are not the only mysterious objects in the theory of general relativity. In an attempt to construct a geometrical model for an elementary particle which excludes singularities, Einstein and Rosen [9] (see also [10]), introduced the concept of a

bridge: a region in spacetime that connects two separate sheets. A simple coordinate transformation that is valid everywhere outside the event horizon of a Schwarzschild black hole is enough to bring the metric into the form of a bridge. In modern terminology, the bridge is referred to as a *wormhole*. But the Schwarzschild wormhole, sometimes called neutral wormhole due to the absence of electromagnetic charges, is dynamical [11], and it pinches off so fast that not even light can cross the throat separating the two sheets. In other words, the neutral bridge is a non-traversable wormhole. Einstein and Rosen [9] also tried to construct a charged bridge by using the RN black hole, as will be shown later, but this violated the null energy condition (NEC) everywhere in the spacetime. Matter that violates the NEC is called exotic matter, and there is no known charged wormhole solution that does not require exotic matter to exist.

In 1988, Morris and Thorne [12] investigated traversable wormholes, which are bridges that would allow a human being to cross them. Although quantum mechanically the energy is allowed to acquire negative values at the throat of a wormhole, all the traversable solutions [13, 14] together with the charged bridge discussed above require the existence of exotic matter for them to exist classically. The need of exotic matter can be circumvented by considering modified theories of gravity, see [15–18] for instance. A possible type of exotic matter can be a phantom field, which is scalar field with negative kinetic energy [19, 20]. Gibbons and Rasheed [21] investigated the relation between these phantom fields and zero mass objects, such as black holes and wormholes. They discussed Dyson pairs production in the context of gravity coupled to vector fields and scalars, and argued that they consist of zero rest mass black holes with regular horizons. Another interesting and more recent idea, known as ER=EPR, suggests that entangled black holes are connected by wormholes.

In this paper we give (as far as we are aware) the

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first zero mass analytical black hole solution that is not made up of pairs of black holes. They arise as the massless limit of the recently found five parameters dyonic black hole solution of the EMD theory [22]. The freedom of imposing boundary conditions on the integration constants of the solution is used to construct massless non-extremal and extremal black holes, as well as massless naked singularities. From the massless black holes we construct charged wormholes, and show that they satisfy the NEC. This is a proof that charged wormhole solutions satisfying the NEC exist, and since the EMD theory can be embedded in string theory, this is also a proof that they are solutions to string theory.

II. DYONIC BLACK HOLE FOR EMD THEORY

We consider the EMD theory without a dilaton potential. The coupling to the field strength is inspired in supergravity models and take the usual exponential form (for the motivation to consider this theory, see [8] and references therein). The action is written as

$$S = \int d^4x \sqrt{-g} (R - 2\partial_\mu \phi \partial^\mu \phi - e^{-2\phi} F_{\mu\nu} F^{\mu\nu}). \quad (1)$$

We take units in which $(16\pi G_N) \equiv 1$, where G_N is the Newton's constant. The field strength has the usual definition, i.e.

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu. \quad (2)$$

The most general dyonic black hole solution takes the form [22]

$$ds^2 = -e^{-\lambda} dt^2 + e^\lambda dr^2 + C^2(r) d\Omega_2^2, \quad (3)$$

$$e^{-\lambda} = \frac{(r-r_1)(r-r_2)}{(r+d_0)(r+d_1)}, \quad C^2(r) = (r+d_0)(r+d_1), \quad (4)$$

$$e^{2\phi} = e^{2\phi_0} \frac{r+d_1}{r+d_0}, \quad (5)$$

$$F_{rt} = \frac{e^{2\phi_0} Q}{(r+d_0)^2}, \quad F_{\theta\phi} = P \sin \theta, \quad (6)$$

with

$$d_0 = \frac{-(r_1+r_2) \pm \sqrt{(r_1-r_2)^2 + 8e^{2\phi_0} Q^2}}{2}, \quad (7)$$

$$d_1 = \frac{-(r_1+r_2) \pm \sqrt{(r_1-r_2)^2 + 8e^{-2\phi_0} P^2}}{2}. \quad (8)$$

This solution contains five independent parameters: the electric charge Q , the magnetic charge P , the value of the dilaton at infinity ϕ_0 , and two integration constants,

r_1 and r_2 . This solution is totally free of boundary conditions, which must be imposed on r_1 and r_2 . The g_{tt} component of the metric have the following expansion far from the black hole

$$g_{tt} = -\left(1 - \frac{(r_1+r_2)}{r}\right) + \mathcal{O}\left(\frac{1}{r^2}\right). \quad (9)$$

In the Newtonian approximation the mass M of the black hole is defined as

$$(r_1+r_2) = 2M. \quad (10)$$

This definition of mass is consistent with the one used in reference [8]. From now on, we will consider only the case for which the black hole is massless, i.e. $(r_1+r_2) = 2M = 0$.

III. MASSLESS POINTLIKE DYONIC SOLUTIONS

In order to analyse the massless pointlike objects of the theory, we rename $r_1 = -r_2 \equiv r_0$. There are three cases:

- Non-extremal dyonic solution:

The non-extremal solution is achieved just by choosing the signs of (7) and (8) to be pluses. This guarantees that the singularities are covered by an event horizon. For $d_0 > d_1$, one is lead to think that the domain of validity of the r coordinate is $r \geq d_0$, since the black hole horizon area shrinks to zero at $r = d_0$. But this is not the case. This coordinate system is valid only outside the outer event horizon r_+ . The metric takes the form

$$ds^2 = -e^{-\lambda} dt^2 + e^\lambda dr^2 + C^2(r) d\Omega_2^2, \\ e^{-\lambda} = \frac{(r^2 - r_0^2)}{(r+d_0)(r+d_1)}, \quad C^2(r) = (r+d_0)(r+d_1), \\ d_0 = \sqrt{r_0^2 + 2e^{2\phi_0} Q^2}, \quad d_1 = \sqrt{r_0^2 + 2e^{-2\phi_0} P^2}. \quad (11)$$

The dilaton and gauge fields are still written as in equations (5) and (6) respectively. The singularity is located at $r_S = -d_0$. The position of the inner horizon r_- , the outer horizon r_+ , and the singularity r_S are given by

$$r_+ = +r_0, \quad r_- = -r_0, \quad r_S = -\sqrt{r_0^2 + 2e^{2\phi_0} Q^2}. \quad (12)$$

There is also a temperature T and an entropy S associated to this object, given by

$$T = \frac{2r_0}{(r_0+d_0)(r_0+d_1)}, \quad S = \pi(r_0+d_0)(r_0+d_1). \quad (13)$$

As one can see, the invariant quantities depend on r_0 , i.e. they depend on the boundary conditions imposed on our solutions.

- Extremal dyonic solution:

Just like the RN solution, the extremal limit is achieved when the two horizons coincide, i.e. $r_0 = 0$. The extremal solution is written in isotropic coordinates, $r^2 = x_1^2 + x_2^2 + x_3^2$, and is given by

$$ds^2 = -e^{-\lambda} dt^2 + e^{\lambda} d\vec{x}^2, \quad (14)$$

$$e^{-\lambda} = \frac{r^2}{(r + \sqrt{2}e^{\phi_0}Q)(r + \sqrt{2}e^{-\phi_0}P)},$$

$$e^{2\phi} = e^{2\phi_0} \frac{r + \sqrt{2}e^{-\phi_0}P}{r + \sqrt{2}e^{\phi_0}Q}. \quad (15)$$

The black hole horizon and singularity are located at

$$r_H = 0, \quad r_S = -\sqrt{2}e^{\phi_0}Q. \quad (16)$$

The temperature and entropy are written as

$$T = 0, \quad S = 2\pi QP. \quad (17)$$

The fact that extremal solutions can also be massless can be easily seen from equation (37) of reference [22].

- Naked singularity:

In order to obtain a naked singularity we have to choose the signs in (7) and (8) to be minuses. For $d_0 > d_1$, this fixes the metric to be valid only in the domain $r > \sqrt{r_0^2 + 2e^{2\phi_0}Q^2}$, so that there is no horizon in this solution. We choose $r_0 = 0$ without loss of generality. The massless naked singularity solution written in isotropic coordinates is given by

$$ds^2 = -e^{-\lambda} dt^2 + e^{\lambda} d\vec{x}^2, \quad (18)$$

$$e^{-\lambda} = \frac{r^2}{(r - \sqrt{2}e^{\phi_0}Q)(r - \sqrt{2}e^{-\phi_0}P)},$$

$$e^{2\phi} = e^{2\phi_0} \frac{r - \sqrt{2}e^{-\phi_0}P}{r - \sqrt{2}e^{\phi_0}Q}.$$

The massless solution in [2] is written as

$$ds^2 = -\left(1 - \frac{D^2}{r^2}\right)^{-1/2} dt^2 + \left(1 - \frac{D^2}{r^2}\right)^{1/2} d\vec{x}^2. \quad (19)$$

A question that is still unanswered is whether this solution represents the extremal limit of a known black hole solution. We see that this is not the case for the extremal dyonic solution and naked singularity obtained from the massless non-extremal solution of EMD theory.

IV. BRIDGE CONSTRUCTION

Consider the RN solution

$$ds^2 = -e^{-\lambda} dt^2 + e^{\lambda} dr^2 + r^2 d\Omega_2^2, \quad (20)$$

$$e^{-\lambda} = 1 - \frac{2M}{r} + \frac{Q^2 + P^2}{r^2},$$

$$F_{rt} = \frac{Q}{r^2}, \quad F_{\theta\phi} = P \sin \theta.$$

By setting the mass M to zero, the solution turns into a naked singularity, i.e. it has no horizon. The existence of a horizon is a necessary condition for the bridge construction, so, Einstein and Rosen [9] considered imaginary charges, such that the sign in front of the term containing the charges squared would be minus. This corresponds to setting $(Q^2 + P^2) \equiv -\epsilon^2$, for a constant ϵ . This solution now has a horizon, and then the coordinate change $u^2 = r^2 - \epsilon^2$ brings the metric into the bridge form

$$ds^2 = -\frac{u^2}{u^2 + \epsilon^2} dt^2 + du^2 + (u^2 + \epsilon^2) d\Omega_2^2. \quad (21)$$

The throat of this charged bridge is at $u = 0$. But the matter in this solution is exotic: it has negative energy density everywhere in space.

We will see that this is not the case for the bridges constructed from the non-extremal and extremal massless solutions of the previous sections. Notice that by switching to the coordinates $u^2 = r^2 - r_0^2$, the metric (11) is written as

$$ds^2 = -\frac{u^2}{(u^2 + r_0^2) f(u)} dt^2 + f(u) du^2 + (u^2 + r_0^2) f(u) d\Omega_2^2, \quad (22)$$

$$f(u) = \left(1 + \frac{d_0}{\sqrt{u^2 + r_0^2}}\right) \left(1 + \frac{d_1}{\sqrt{u^2 + r_0^2}}\right).$$

This is a genuine charged wormhole solution: it connects one Minkowski space at $u = -\infty$ to another at $u = +\infty$. The throat of the wormhole is located at $u = 0$, and it has radius

$$R_{\text{throat}} = \sqrt{(r_0 + d_0)(r_0 + d_1)}, \quad (23)$$

where d_0 and d_1 are given in (11). The term inside the square root is always positive, and the throat of the wormhole will always be greater than zero for $|Q|, |P| > 0$. Notice that this solution is valid only outside the horizon, $r > +r_0$. For $r_0 = 0$, the coordinate r is given by $r = |u|$, and the solution is written as

$$ds^2 = -\frac{|u|^2}{(|u| + d_0)(|u| + d_1)} dt^2 + \frac{(|u| + d_0)(|u| + d_1)}{|u|^2} d\vec{x}^2, \quad (24)$$

where we have used isotropic coordinates $u^2 = x_1^2 + x_2^2 + x_3^2$. The radius of the throat now will be

$$R_{\text{throat}} = \sqrt{2QP}. \quad (25)$$

We see that the charged wormholes in the EMD theory may come from the extremal and non-extremal dyonic solutions. We will discuss later the physical consequences of this observation.

V. NULL ENERGY CONDITION

Unlike the charged wormholes constructed from the RN solution, we did not need to consider imaginary charges to achieve (22) and (24). This gives some hope that these charged wormholes do not require exotic matter to exist. As stated before, exotic matter violates the NEC, and we now check whether this is the case in the present paper or not. We will use the original r coordinate, since the null energy condition does not depend on the choice of coordinate systems. We follow the same prescription as in [12], and prove our statements for the non-extremal massless solution. The Ricci tensors for the metric (11) are then

$$R_{tt} = \frac{(r^2 - r_0^2)}{2(d_0 + r)^4(d_1 + r)^4} [d_0^2(2d_1^2 + 2d_1r + r^2 - r_0^2) + 2d_0r(d_1^2 - r_0^2) + d_1^2(r^2 - r_0^2) - 2d_1rr_0^2 - 2r^2r_0^2], \quad (26)$$

$$R_{rr} = \frac{r_0^2 - d_0d_1}{(d_0 + r)(d_1 + r)(r^2 - r_0^2)}, \quad (27)$$

$$R_{\theta\theta} = \frac{1}{2(d_0 + r)^2(d_1 + r)^2} [d_0^2(2d_1^2 + 2d_1r + r^2 - r_0^2) + 2d_0r(d_1^2 - r_0^2) + d_1^2(r^2 - r_0^2) - 2d_1rr_0^2 - 2r^2r_0^2], \quad (28)$$

$$R_{\phi\phi} = R_{\theta\theta} \sin^2 \theta. \quad (29)$$

The curvature tensor is

$$R = \frac{(d_0 - d_1)^2(r^2 - r_0^2)}{2(d_0 + r)^3(d_1 + r)^3}. \quad (30)$$

We choose orthonormal basis vectors [12]:

$$\mathbf{e}_t = \left(\frac{(r + d_0)(r + d_1)}{(r^2 - r_0^2)} \right)^{1/2} \mathbf{e}_t, \quad (31)$$

$$\mathbf{e}_{\hat{r}} = \left(\frac{(r^2 - r_0^2)}{(r + d_0)(r + d_1)} \right)^{1/2} \mathbf{e}_r, \quad (32)$$

$$\mathbf{e}_{\hat{\theta}} = \left(\frac{1}{(r + d_0)(r + d_1)} \right)^{1/2} \mathbf{e}_{\theta}, \quad (33)$$

$$\mathbf{e}_{\hat{\phi}} = \left(\frac{1}{(r + d_0)(r + d_1)} \right)^{1/2} \frac{1}{\sin \theta} \mathbf{e}_{\phi}. \quad (34)$$

In this basis the metric coefficients take the form $\mathbf{g}_{\hat{\alpha}\hat{\beta}} = \mathbf{e}_{\hat{\alpha}} \cdot \mathbf{e}_{\hat{\beta}} = \eta_{\hat{\alpha}\hat{\beta}} = \text{diag}(-1, 1, 1, 1)$. Einstein's equations take the form

$$G_{\hat{\mu}\hat{\nu}} = 8\pi G_N T_{\hat{\mu}\hat{\nu}}. \quad (35)$$

In our unities $(16\pi G_N) = 1$. The components of the energy momentum tensor are $T_{\hat{t}\hat{t}} = \rho(r)$, $T_{\hat{r}\hat{r}} = -\tau(r)$, $T_{\hat{\theta}\hat{\theta}} = T_{\hat{\phi}\hat{\phi}} = p(r)$, where $\rho(r)$ is the energy density measured by the static observer, $\tau(r)$ is the tension per unit area measured in the radial direction, and $p(r)$ is the pressure that is measured in the directions orthogonal to the radial direction. They are given by

$$\begin{aligned} \rho(r) &= \frac{1}{2(d_0 + r)^3(d_1 + r)^3} [4d_0r(d_1^2 - r_0^2) \\ &\quad + 2d_0^2(2d_1^2 + 2d_1r + r^2 - r_0^2) + 2d_1^2(r^2 - r_0^2) \\ &\quad - 4d_1rr_0^2 - 4r^2r_0^2 + (d_0 - d_1)^2(r^2 - r_0^2)], \quad (36) \\ -\tau(r) &= \frac{1}{2(d_0 + r)^3(d_1 + r)^3} [-2(d_0 - d_1)^2(r^2 - r_0^2) \\ &\quad + (4r_0^2 - 4d_0d_1)(d_0 + r)(d_1 + r)]. \quad (37) \end{aligned}$$

The NEC states that

$$T_{\hat{\mu}\hat{\nu}}k^{\hat{\mu}}k^{\hat{\nu}} \geq 0. \quad (38)$$

In the same coordinate system, the null vector is given by $k^{\hat{\mu}} = (1, 1, 0, 0)$, and the NEC results in

$$\rho(r) - \tau(r) = \frac{(d_0 - d_1)^2(r^2 - r_0^2)}{2(d_0 + r)^3(d_1 + r)^3} \geq 0. \quad (39)$$

Notice that the bound is saturated at the throat of the wormhole, i.e.

$$\rho(r_0) - \tau(r_0) = 0. \quad (40)$$

This shows that the NEC is satisfied, and the charged wormhole solution presented here does not require exotic matter to exist. The NEC is still satisfied for $r_0 = 0$, i.e. for the bridge built from the massless extremal dyonic solution (14).

The massless pointlike objects and charged wormholes presented here are entirely new, and, of course, there are many directions in which one may follow to answer the questions that may have arisen in the text. One should, for instance, try to understand the physical process that makes the massless non-extremal black hole becomes extremal (or, that makes the non-extremal bridge becomes extremal). This can be done by choosing the independent parameter r_0 to depend on the remaining constants, Q , P and ϕ_0 . But notice that this will be a choice, since the equations of motion only do not require it to depend on the other parameter in any specific functional form. This will also help us to understand the thermodynamical evolution of the massless solutions and charged bridges. Another immediate question concerns stability, since a full analysis is necessary to prove that our solutions are stable under small perturbations of the metric. Everything we discussed is independent of supersymmetry, but we can add it as an extra ingredient to the analysis. If we do so, we can ask what are the conditions that should be imposed on our extremal solutions to make them also supersymmetric? Probably, the most important question at this point is whether or not the same argument concerning the traversability of the neutral bridges applies here. Are these charged wormholes traversable? It is mathematically acceptable to set the mass of the black hole to

zero from the beginning, since this is a solution. But, is this physically reasonable? What could constrain the boundary conditions imposed on r_1 and r_2 ? As a matter of fact, the answers to these and many other questions will certainly enrich our knowledge about black holes, not only in string theory but, hopefully, in general.

VI. CONCLUSIONS

In this paper we presented the massless pointlike objects arising as solutions to the EMD theory. They can be non-extremal, extremal and a naked singularity. This shows that massless black holes exist individually, without the need of coming in pairs. From the non-extremal solution, we constructed a charged bridge and showed that this satisfies the NEC, a result that is also valid in the extremal limit.

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